2(i) V.V.Q. 1st copy C+iS method.

D1(H) Maths paper-1st, Group-A summation of series by C+iS method.

3(i) 1st copy

3(ii) $C = \cos\theta + \frac{\cos\theta}{\lfloor 1}\cos 2\theta + \frac{\cos^2\theta}{\lfloor 2}\cos 3\theta + \cdots$

$S = \sin\theta + \frac{\cos\theta}{\lfloor 1}\sin 2\theta + \frac{\cos^2\theta}{\lfloor 2}\sin 3\theta + \cdots$

$C+iS = (\cos\theta + i\sin\theta) + \frac{\cos\theta}{\lfloor 1}(\cos 2\theta + i\sin 2\theta) + \frac{\cos^2\theta}{\lfloor 2}(\cos 3\theta + i\sin 3\theta)$

3(iii) $S = \sin\theta\cos\theta + \sin 2\theta\,\frac{\cos^2\theta}{\lfloor 2} + \frac{\cos^3\theta\cdot\sin 3\theta}{\lfloor 3} + \cdots$

$C = 1 + \cos\theta\cdot\cos\theta + \frac{\cos^2\theta\cdot\cos 2\theta}{\lfloor 2} + \frac{\cos^3\theta\cdot\cos 3\theta}{\lfloor 3} + \cdots$

$\therefore C+iS = 1 + \cos\theta(\cos\theta + i\sin\theta) + \frac{\cos^2\theta}{\lfloor 2}(\cos 2\theta + i\sin 2\theta)$

$= 1 + \cos\theta\, e^{i\theta} + \frac{\cos^2\theta}{\lfloor 2}e^{2i\theta} + \cdots$

$= 1 + x + \frac{x^2}{\lfloor 2} + \cdots = e^x = e^{\cos\theta\cdot e^{i\theta}}$

3.(viii) $C = 1 + \cos\alpha\cos\beta + \frac{\cos^2\alpha}{\lfloor 2}\cos 2\beta + \cdots$

$S = \cos\alpha\sin\beta + \frac{\cos^2\alpha}{\lfloor 2}\sin 2\beta + \cdots$

$\therefore C+iS = 1 + \cos\alpha(\cos\beta + i\sin\beta) + \frac{\cos^2\alpha}{\lfloor 2}(\cos 2\beta + i\sin 2\beta) + \cdots$

$= 1 + \cos\alpha\, e^{i\beta} + \frac{\cos^2\alpha}{\lfloor 2}e^{i2\beta} + \frac{\cos^3\alpha}{\lfloor 3}e^{i3\beta} + \cdots$

Let $x = e^{i\beta}$

$= 1 + \cos\alpha\, x + \frac{\cos^2\alpha}{\lfloor 2}x^2 + \frac{\cos^3\alpha}{\lfloor 3}x^3 + \cdots$

$= e^{x\cos\alpha} = e^{e^{i\beta}\cos\alpha} = e^{(\cos\beta + i\sin\beta)\cos\alpha}$ ②

$= e^{\cos\alpha\cos\beta + i\cos\alpha\sin\beta}$

$= e^{\cos\alpha\cos\beta} \cdot e^{i\cos\alpha\sin\beta}$

$C + iS = e^{\cos\alpha\cos\beta}\{\cos(\cos\alpha\sin\beta) + i\sin(\cos\alpha\sin\beta)\}$

$\therefore C = e^{\cos\alpha\cos\beta}\cos(\cos\alpha\sin\beta)$ and ~~$S = e^{\cos\alpha\sin\beta}$~~

and $S = e^{\cos\alpha\cos\beta}\sin(\cos\alpha\sin\beta)$

3(iv) $C = \cos\theta + \frac{\operatorname{cosec}\theta}{\lfloor 1}\cos 2\theta + \frac{\operatorname{cosec}^2\theta\cos 3\theta}{\lfloor 2} + \cdots$

$S = \sin\theta + \frac{\operatorname{cosec}\theta\sin 2\theta}{\lfloor 1} + \operatorname{cosec}^2\theta\frac{\sin 3\theta}{\lfloor 2}$

$\therefore C + iS = (\cos\theta + i\sin\theta) + \frac{\operatorname{cosec}\theta}{\lfloor 1}(\cos 2\theta + i\sin 2\theta)$

$+ \frac{\operatorname{cosec}^2\theta}{\lfloor 2}(\cos 3\theta + i\sin 3\theta) + \cdots$

$= e^{i\theta} + \frac{\operatorname{cosec}\theta}{\lfloor 1}e^{i2\theta} + \frac{\operatorname{cosec}^2\theta}{\lfloor 2}e^{3i\theta} + \cdots$

$= e^{i\theta}\{1 + \frac{\operatorname{cosec}\theta}{\lfloor 1}e^{i\theta} + \frac{\operatorname{cosec}^2\theta}{\lfloor 2}e^{2i\theta} + \cdots\}$

Let $e^{i\theta}\operatorname{cosec}\theta = x$ or let $x = \operatorname{cosec} e^{i\theta}$

$\therefore C + iS = e^{i\theta}\{1 + \frac{\operatorname{cosec}\theta}{\lfloor 1}e^{i\theta} + \frac{\operatorname{cosec}^2\theta}{\lfloor 2}e^{2i\theta} + \cdots\}$

Let $x = \operatorname{cosec}\theta\, e^{i\theta}$

$= e^{i\theta}\{1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots\}$

$= e^{i\theta}\{e^x\}$

$= e^{x + i\theta} = e^{\operatorname{cosec}\theta e^{i\theta} + i\theta}$

$= e^{\operatorname{cosec}\theta(\cos\theta + i\sin\theta) + i\theta}$

$= e^{\operatorname{cosec}\theta\cos\theta + i\operatorname{cosec}\theta\sin\theta + i\theta}$

$= e^{\cot\theta + i + i\theta} = e^{\cot\theta + i(1+\theta)}$

$= e^{\cot\theta}e^{i(1+\theta)}$

$= e^{\cot\theta}\{\cos(1+\theta) + i\sin(1+\theta)\}$

$\therefore C = e^{\cot\theta}\cos(1+\theta)$

3(v) $C = \cos\theta + \frac{\sin\theta}{\lfloor 1}\cos 2\theta + \frac{\sin^2\theta}{\lfloor 2}\cos 3\theta + \cdots$

$S = \sin\theta + \frac{\sin\theta}{\lfloor 1}\sin 2\theta + \frac{\sin^2\theta}{\lfloor 2}\sin 3\theta + \cdots$

$\therefore C + iS = (\cos\theta + i\sin\theta) + \frac{\sin\theta}{\lfloor 1}(\cos 2\theta + i\sin 2\theta) + \frac{\sin^2\theta}{\lfloor 2}(\cos 3\theta + i\sin 3\theta)$

$= e^{i\theta} + \frac{\sin\theta}{\lfloor 1} e^{i2\theta} + \frac{\sin^2\theta}{\lfloor 2} e^{i3\theta} + \cdots$

$e^{i\theta} = x$ ধরি

$\therefore C + iS = x + \frac{\sin\theta}{\lfloor 1} x^2 + \frac{\sin^2\theta}{\lfloor 2} x^3 + \cdots$

$= x\left[1 + \frac{x\sin\theta}{\lfloor 1} + \frac{(x\sin\theta)^2}{\lfloor 2} + \cdots\right]$

$= x\, e^{x\sin\theta} = e^{i\theta} \cdot e^{i\theta\sin\theta} = e^{i\theta} e^{(\cos\theta + i\sin\theta)\sin\theta}$

$= e^{i\theta + \sin\theta\cos\theta + i\sin^2\theta}$

$= e^{\sin\theta\cos\theta + i(\theta + \sin^2\theta)}$

$= e^{\sin\theta\cos\theta} \cdot e^{i(\theta + \sin^2\theta)}$

$= e^{\sin\theta\cos\theta}\{\cos(\theta + \sin^2\theta) + i\sin(\theta + \sin^2\theta)\}$

$\therefore C = e^{\sin\theta\cos\theta} \cdot \cos(\theta + \sin^2\theta)$

and $S = e^{\sin\theta\cos\theta}\sin(\theta + \sin^2\theta)$